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How Strong is the Evidence for Electroweak Corrections Beyond the Running of α ?

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ABSTRACT

The evidence for electroweak radiative corrections not contained in $\alpha(m_Z)$ is examined. At low energies there is very strong evidence in the analysis of the unitarity of the CKM matrix. At LEP and collider experiments the current direct signals are not strong, but there is substantial indirect evidence, which will likely become sharper when m_t is determined. The advantage of using $(\Delta r)_{res}$ as a measure of these effects is emphasized. In order to improve the direct evidence, more accurate measurements of m_W and the on-resonance asymmetries are indicated.

The presence in electroweak physics of large corrections associated with the running of α at the vector boson scale was emphasized long ago [1]. The detailed analysis of electroweak corrections not described by the running of α is also a matter of long standing among particle physicists [2–5]. Recent discussions of signals for the latter effects have focussed on LEP and collider studies [6, 7]. On the basis of the directly measured values of m_W and m_Z , Hioki concluded that there is some evidence for their existence, but only at the 1σ level [6]. Very recently, Novikov, Okun and Vysotsky (N–O–V) pointed out that a Born approximation treatment based on $\alpha(m_Z)$ and a suitable definition of the weak mixing angle reproduces very well the most precise LEP and collider information [7]. From this observation they concluded that there is no evidence for corrections beyond those associated with $\alpha(m_Z)$. In their formulation, sharp constraints on m_t emerge, as in many previous analyses of electroweak data. However, they are interpreted as stemming from the cancellation of top quark effects against those of other virtual particles.

The aim of this paper is to examine the evidence for electroweak corrections not described by the running of α at the m_Z scale. For reasons that will become clear later, we generically denote this parameter as α_{run} . We first consider precision experiments at very low energies and then turn our attention to LEP, SLC and collider physics.

Very strong evidence for electroweak corrections not involving α_{run} is found in the analysis of the universality of the weak interactions [2]. It is well known that the Standard Model (SM) leads to the relation $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$. This is simply a consequence of the unitarity of the CKM matrix and indeed constitutes one of the most fundamental predictions of the theory. We recall that $|V_{ud}|^2$ is determined from the ratio of transition rates of superallowed Fermi transitions and μ decay, while V_{us} and V_{ub} involve consideration of $\Delta S = 1$ and B decays. As an example, when ^{14}O is employed and appropriate electroweak and nuclear overlap corrections are applied, one currently finds [8] $V_{ud} = 0.9745 \pm 0.0005 \pm 0.0004$. The first error includes statistical and nuclear overlap uncertainties, while the second stems from the radiative corrections. In conjunction with V_{us} and V_{ub} this leads to [8]

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9983 \pm 0.0015 \quad (^{14}\text{O}), \quad (1)$$

which is in good agreement with the theoretical expectations. It is important to note, however, that the electroweak corrections applied in obtaining $|V_{ud}|^2$ are very large, namely 4.1% in the case of ^{14}O . As a consequence, if these

corrections and corresponding effects in $|V_{us}|^2$ are not included, Eq.(1) becomes

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1.0386 \pm 0.0013, \quad (2)$$

which differs from unity by ≈ 30 times the estimated error. If the average of the eight accurately measured superallowed Fermi transitions is employed rather than ^{14}O , there are small changes at the 0.2% level in Eq.(1), but the sharp disagreement illustrated in Eq.(2) still holds. Thus, we see that the large electroweak corrections obtained in the SM are actually crucial for its survival as a viable theory. Because these effects cannot be computed consistently in the local Fermi theory, where they are divergent, their existence and the close agreement that follows from their application can be regarded as a significant success of the SM. We also note that β , μ and semileptonic decays occur at $q^2 \approx 0$ and that these large corrections are not associated with the running of α .

In order to discuss the evidence at LEP and collider physics one must first precisely state how α_{run} is defined. This concept is scheme-dependent and, in fact, there are two frequently employed definitions,

$$\alpha(m_Z) = \alpha/[1 + e^2 \text{Re}\Pi_{\gamma\gamma}^r(m_Z^2)], \quad (3)$$

and

$$\hat{\alpha}(m_Z) = \alpha/[1 - e^2 \Pi_{\gamma\gamma}(0)|_{\overline{\text{MS}}} + \dots]. \quad (4)$$

Here $\Pi_{\gamma\gamma}(q^2)$ is the unrenormalized vacuum polarization function and the ellipses in Eq.(4) stand for additional contributions involving W 's. The superscript r in Eq.(3) indicates the conventional QED renormalization, namely the subtraction of $\Pi_{\gamma\gamma}(0)$, while $\overline{\text{MS}}$ in Eq.(4) denotes the $\overline{\text{MS}}$ renormalization. In the latter case one subtracts the poles and associated constants in dimensional regularization and chooses the 't-Hooft mass scale μ to be equal to m_Z . Both definitions absorb the large logarithms associated with the running of α , but there is a 0.8% numerical difference between them. Specifically, $\alpha(m_Z) = (128.87 \pm 0.12)^{-1}$ [9] and $\hat{\alpha}(m_Z) = (127.9 \pm 0.1)^{-1}$ [10].

Employing Eq.(3) and defining

$$c_0^2 s_0^2 = \pi\alpha(m_Z)/[\sqrt{2}G_\mu m_Z^2], \quad (5)$$

so that $s_0^2 = 0.23118(31)$, N-O-V found that, at the time of their writing, the data for m_W/m_Z , $(g_A)_\ell$, $(g_V/g_A)_\ell$, Γ_ℓ , Γ_h and Γ_Z were reproduced within their

1σ accuracies by a simple Born approximation (B. A.) calculation [7]. Here $(g_A)_\ell$ and $(g_V)_\ell$ are the effective couplings of Z^0 to leptons at $q^2 = m_Z^2$, while Γ_ℓ , Γ_h and Γ_Z represent the leptonic, hadronic and total widths of Z^0 . In particular, the predictions for Γ_ℓ , Γ_h and Γ_Z are remarkably accurate and the corresponding experimental values quite stable. On the other hand, $(g_V/g_A)_\ell$ is very sensitive to small variations in the electroweak data. For instance, a more recent analysis which includes the preliminary high statistics 1992 LEP run, leads to $\sin^2 \theta_{eff}^{lept} = 0.2321 \pm 0.0006$, as determined from the on-resonance asymmetries [11]. This translates into $(g_V/g_A)_\ell = 1 - 4 \sin^2 \theta_{eff}^{lept} = 0.0716(24)$, which differs from the B.A. prediction by 1.4σ . Also, in the determination of m_W/m_Z N-O-V employed only the direct collider data. If one also includes the value $\sin^2 \theta_W \equiv 1 - m_W^2/m_Z^2 = 0.2257 \pm 0.0046$, obtained from νN scattering [12], one finds $m_W/m_Z = 0.8799 \pm 0.0019$, which differs from the B.A. by 1.6σ . We also note that if we were to use $\hat{\alpha}(m_Z)$ instead of $\alpha(m_Z)$ in Eq.(5), we would obtain $\hat{s}_0^2 = 0.2337(3)$ and the corresponding B.A. predictions for $(g_V/g_A)_\ell$ and m_W/m_Z would differ from the experimental values by 2.4σ . However, in this alternative B.A. formulation, Γ_ℓ , Γ_h , and Γ_Z are still acceptable, roughly at the 1σ level.

Thus, we see that in the most favorable of the two B.A. approaches, namely the $\alpha(m_Z)$, s_0^2 scheme proposed by N-O-V, the current direct evidence for additional corrections involves deviations of $\approx 1.4\sigma$ in $(g_V/g_A)_\ell$ and $\approx 1.6\sigma$ in m_W/m_Z if the νN determination is included, and is therefore not strong. However, such signals are likely to be very volatile in the short run, as they depend sensitively on the central values for $\sin^2 \theta_{eff}^{lept}$ and m_W/m_Z , and their corresponding errors.

On the other hand, it is important to emphasize that there is at present substantial indirect or inferred evidence from LEP and collider physics for significant corrections beyond α_{run} . Such evidence can be uncovered by analyzing the various observables in the framework of the complete theory, including its complex panoply of electroweak corrections and interlocking relations [13]. The reason is that, under such scrutiny, the SM becomes highly constrained. In particular, the very recent analysis of Ref.[11] leads to $m_t = 164_{-17}^{+16} {}_{-21}^{+18}$ GeV (the central value is for $m_H = 300$ GeV, while the second uncertainty reflects the shifts corresponding to $m_H = 60$ GeV and 1 TeV). As a heavy top quark decouples from $\alpha(m_Z)$, it is clear that such constraint arises from the study of electroweak corrections not contained in $\alpha(m_Z)$. The same observation applies to the search for signals of new physics in quantum loop effects [14]. A particularly beautiful illustration of how one can obtain strong indirect evidence for

such corrections is provided by the analysis of m_W . There are at present three independent ways to determine this fundamental parameter: i) using m_W from CDF, m_W/m_Z and m_W from UA2, and $m_Z - m_W$ from UA1, in conjunction with the LEP value for m_Z , one finds $m_W = 80.23 \pm 0.26$ GeV ii) employing $\sin^2 \theta_W = 0.2257 \pm 0.0046$ from νN scattering [12], one has $m_W = 80.24 \pm 0.24$ GeV iii) from the LEP observables, via the electroweak corrections, one obtains the indirect determination $m_W = 80.25 \pm 0.10^{+0.02}_{-0.03}$ GeV [11]. The consistency of the three values and the accuracy of iii) are quite remarkable. Choosing in iii) $m_W = 80.22 \pm 0.10$ GeV, corresponding to $m_H = 60$ GeV (the most unfavorable option for our argument), the weighted average of the three values becomes $m_W = 80.22 \pm 0.087$ GeV. This differs from $m_W = 79.95 \pm 0.02$ GeV (the N–O–V B.A. prediction) by 3σ and from $m_W = 79.82 \pm 0.02$ GeV (the $\hat{\alpha}(m_Z)$, \hat{s}_0^2 B.A. prediction) by 4.5σ .

This result can also be expressed in terms of $(\Delta r)_{\text{res}}$ [10, 15], the residual part of Δr after extracting the effects associated with the running of α . Recalling the relation [4]

$$m_W^2 \left(1 - \frac{m_W^2}{m_Z^2} \right) = \frac{\pi\alpha}{\sqrt{2}G_\mu(1 - \Delta r)}, \quad (6)$$

we see that, given α , G_μ , and m_Z , a determination of m_W leads to a definite value for Δr . In particular, using $m_W = 80.22 \pm 0.087$ GeV one finds $\Delta r = 0.0447 \pm 0.0051$. Writing

$$\alpha/(1 - \Delta r) = \alpha_{\text{run}}/(1 - (\Delta r)_{\text{res}}), \quad (7)$$

one has $(\Delta r)_{\text{res}} = -0.0158 \pm 0.0054$ if $\alpha_{\text{run}} = \alpha(m_Z)$ (Cf. Eq.(3)) and $(\Delta r)_{\text{res}} = -0.0235 \pm 0.0055$ if $\alpha_{\text{run}} = \hat{\alpha}(m_Z)$ (Cf. Eq.(4)). We see quite clearly that $(\Delta r)_{\text{res}}$ is not zero but differs from a null result by $\approx 2.9\sigma$ or $\approx 4.3\sigma$, depending on how the running of α is parametrized. Recalling that the natural dimensionless coupling for electroweak corrections not contained in α_{run} is $\hat{\alpha}/2\pi\hat{s}^2 \approx 0.54 \times 10^{-2}$ ($\hat{s}^2 \equiv \sin^2 \hat{\theta}_W(m_Z)$ is the $\overline{\text{MS}}$ parameter), we also see that the central values for $(\Delta r)_{\text{res}}$ given above are not small. Thus, the current global analysis of LEP and collider physics, based on the complete theory, points out to the existence of significant electroweak corrections beyond α_{run} . There is another important theoretical advantage in using $(\Delta r)_{\text{res}}$ as a signal for corrections “beyond the running of α ”. The point is that, as illustrated in Eq.(6), Δr is a physical observable. This means that Δr is renormalization–scheme independent and, therefore, it is not affected by the way in which the weak mixing angle

is introduced. For instance, if carried out with sufficient accuracy, theoretical calculations of Δr should give the same result whether one identifies the weak mixing parameter with $\sin^2 \hat{\theta}_W(m_Z)$, $\sin^2 \theta_{eff}^{lept}$, s_0^2 , \hat{s}_0^2 , or $1 - m_W^2/m_Z^2$. On the other hand, $(\Delta r)_{res}$ does depend on how α_{run} is defined. This latter ambiguity is unavoidable, as the analysis of the corrections not contained in α_{run} obviously depends on the meaning of this parameter.

Accepting the results of the global analyses in the framework of the complete theory, it is also not difficult to show that no B.A., whether related to α_{run} or not, can accurately describe all the available information. We note the current global values $\sin^2 \theta_{eff}^{lept} = 0.2325 \pm 0.0005^{+0.0001}_{-0.0002}$, $\sin^2 \theta_W = 0.2257 \pm 0.0017^{+0.0003}_{-0.0004}$ [11]. Assuming again $m_H = 60$ GeV, the most unfavorable option for our argument, we see that the two quantities differ by 3.5σ . Furthermore, both are physical observables. Thus, it is clear that no B.A. involving a single s^2 parameter can satisfactorily accomodate the values associated with the two observables.

If the top quark is discovered and its mass measured, the indirect evidence may become much sharper. For instance, if m_t is measured to ± 10 GeV and the corresponding central m_W value remains unaltered, the indirect LEP determination would become approximately $m_W = 80.25 \pm 0.06^{+0.02}_{-0.03}$ GeV. The combined result for $m_H = 60$ GeV (the most unfavorable case) would then be $m_W = 80.22 \pm 0.06$ GeV, which differs from the N-O-V B.A. prediction by 4.3σ .

In order to obtain stronger direct evidence one would like to improve the measurement of important observables such as m_W and $\sin^2 \theta_{eff}^{lept}$. If, for example, the central value of m_W remains at ≈ 80.24 GeV (the value favored by current analyses), a measurement to ± 100 MeV would imply a difference of 2.8σ with the B.A. prediction. It is also important to improve the direct measurement of $\sin^2 \theta_{eff}^{lept}$ from the on-resonance asymmetries at LEP and SLC. An increase in the central value or a decrease in the error would imply a sharper discrepancy with the B.A. calculation. This would be the case, for instance, if the central value derived from the asymmetries (currently 0.2321) shifts in the future towards 0.2325 (the current global value).

In summary, in the analysis of universality [2, 8] there is very strong evidence for the existence of electroweak corrections not contained in α_{run} . In the study of LEP and collider physics we have made a distinction between direct and indirect or inferred evidence. In the first case, important observables such as m_W , $\sin^2 \theta_{eff}^{lept}$, Γ_ℓ , Γ_h , and Γ_Z are determined directly or almost directly from experiments and then compared with predictions of B.A. schemes involving $\alpha(m_Z)$ or $\hat{\alpha}(m_Z)$. This is the approach followed in Ref. [6, 7]. In

the second case precise determinations of fundamental parameters such as m_W , Δr , $\sin^2 \hat{\theta}_W(m_Z)$... are made by analyzing the global information in terms of the complete SM, including its radiative corrections. We have emphasized that $(\Delta r)_{\text{res}}$ provides an important signal independent of the definition of the weak mixing angle. In both cases the evidence depends sensitively on whether one employs $\alpha(m_Z)$ or $\hat{\alpha}(m_Z)$ as parametrizations of the running of α . At LEP and collider experiments the current direct signals for corrections beyond α_{run} are not strong, but there is substantial indirect evidence, which will likely become sharper when m_t is determined. We have also pointed out that there is at present considerable indirect evidence that no B.A. can accurately describe all the available information. Furthermore, the study of such corrections is very important in order to constrain m_t and search for signals of new physics. In order to improve the direct evidence, more accurate measurements of m_W and $\sin^2 \theta_{eff}^{lept}$ are called for.

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